THE FUNDAMENTAL CONCEPTS OF ROBUST COMPLIANT MOTION FOR ROBOT MANIPULATORS

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1. Abstract

Manipulators are subject to interaction forces when they maneuver in a constrained work-space. Our goal is to develop a method for the design of controllers of constrained manipulators in the presence of model uncertainties. The controller must carry out fine maneuvers when the manipulator is not constrained, and compliant motion, with or without interaction-force measurement, when the manipulator is constrained. At the same time stability must be preserved if bounded uncertainties are allowed in modelling the manipulators. Stability of the manipulator and environment as a whole and the preservation of stability in the face of changes are two fundamental issues that have been considered in the design method. We start with conventional controller-design specifications concerning the treatment of external forces when the manipulator is not constrained. Generalizing this concept to include cases when the manipulator is constrained, we state a set of practical design specifications in the frequency domain that is meaningful from the standpoint of control theory and assures the desired compliant motion in the cartesian coordinate frame and stability in the presence of bounded uncertainties. This approach also assures the global stability of the manipulator and its environment. While this paper concerns the fundamentals of compliant motion, part 2 of this paper (Reference 24) is devoted to the controller design method.

2. Introduction

For a broad class of manipulators under closed-loop control, fundamental differences in behavior and controller design complexity can be attributed to two types of maneuvers: unconstrained and constrained. In the first case, the manipulator is driven in its work-space without contact with the environment. Note that the environment might exist in the manipulator work-space without imposing any constraint on the manipulator motion. In constrained maneuvers, the manipulator is driven in its work-space so that the environment continuously exerts a dynamic or kinematic constraint on the manipulator's motion. Spray painting by a manipulator is an example of the first class of maneuver. The end-point of the manipulator travels through certain points in its work-space without any restriction. On the other hand, inserting a computer board in a slot (i.e., the peg-in-hole problem) or turning a crank by means of a manipulator are examples of constrained maneuvers; the end-point of the manipulator is in contact with the environment and cannot move in all directions. A dynamic maneuver such as leading a manipulator in a free environment toward a metal surface and then grinding the surface may consist of both types of maneuvers. Our classification of maneuvers as unconstrained and constrained is similar to the classification introduced by Whitney (20), who categorized manipulations into "rearrangement" tasks and "force" tasks. This paper deals with constrained maneuvering.

In constrained maneuvering, the interaction forces must be accommodated rather than resisted. If we define "compliancy" as a measure of the ability of a manipulator to react to interaction forces and torques, we can state our objective as: to assure compliant motion in the global cartesian coordinate frame for manipulators that must maneuver in constrained environments. Previous researchers have suggested two approaches for assuring compliant motion for manipulators. The first approach is aimed at controlling force (torque) and position (orientation) in a non-conflicting way. In this method, force (torque) is commanded along (about) those directions constrained by the environment, while position (orientation) is commanded along (about) those directions in which the manipulator is unconstrained and free to move. The second approach is aimed at developing a relationship between interaction forces and manipulator position. By controlling the manipulator position and specifying its relationship to the interaction forces, a designer can ensure that the manipulator will be able to maneuver in a constrained environment while maintaining appropriate contact forces.

The first approach was motivated by several studies. Paul and Shimano (16) partitioned the motion of a manipulator into position- and force-control in a global cartesian coordinate frame. Then, with the help of a decision-making "logic" hidden in a supervisory computer program, they arrived at the two sets of actuators that could best contribute to the position control loop and the force control loop. Raibert and Craig (17) also partitioned the motion of the manipulator in a global cartesian coordinate frame. They used a position controller to move the manipulator in unconstrained directions and a force controller to push the manipulator against the environment with the desired contact force. They then arrived at input values for the actuators (without assuring stability) such that all actuators would contribute to both partitions. Whitney (20) arrived at a single-loop velocity-control scheme with the net effect of controlling

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the contact force. Similar work in the generation of compliant motion has been done by Mason (15), and Wu and Paul (23). Common to all such methods for ensuring compliant motion is the dependence of the controller's structure on both the kinematics and dynamics of the manipulator and of its environment. For example, if the end-point of a manipulator travels from one constrained point to another such that the environment at the new point exerts constraints that differ from the constraints at the first point, then a new controller with a different structure must be designed to accommodate the new constraints. In the second approach toward generating compliant motion, a relationship is defined between the position of the manipulator and the interaction forces. Salisbury (19) started by defining a linear static function that relates interaction forces to end-point position via a stiffness matrix in a cartesian coordinate frame. Monitoring this relationship by means of a computer program ensures that the manipulator will be able to maneuver successfully in a constrained environment. In his seminal work, Salisbury justified the stiffness matrix as the representative of a behavior that manipulators must exhibit while they are used as positioning systems. The method of stiffness control offers neither assurance of global dynamic stability nor a guarantee of a specified frequency range of operation.

This paper addresses the problem of closed-loop control of manipulators, that operate in constrained environments, with or without interaction force measurement, in the presence of bounded model uncertainties. Central to the approach is the notion of mechanical impedance (5-10) in frequency domain as a parameterization of a rational set of peformance specifications to generate the compliant motion while preserving stability in the presence of bounded model uncertainties. Preservation of the stability of the manipulator and the environment taken together as a whole is also a fundamental issue in this design method. Our design method is an in-depth frequency-domain approach of Salisbury's stiffness control; therefore, it is considered to be part of the second approach toward developing compliant motion.

3. Definition of Compliant Motion in Control Theory

In this section, we explain (without getting involved in mathematics and design methodologies) points of practical importance in generating the compliant motion of a manipulator. We start with conventional controller-design specifications concerning the accommodation of interaction forces when the manipulator is not constrained. Then we generalize this concept to apply to situations in which the manipulator is constrained. This will lead us to parameterize the necessary performance specifications in a simple mathematical form in frequency domain.

For the classes of manipulators that are used as positioning systems, control compensators traditionally have been designed so the system's outputs (position and orientation) follow the commands, while rejecting the external forces. The two specifications (command-following and external-force rejection) typically require large loop gains for the frequency range in which the command input and the external forces contain the most power. Since commands and external forces usually contain low-frequency signals, command-following and external-force rejection properties taken together establish a design specification at low frequencies. To achieve the above properties over a large frequency range is not trivial; loop gains cannot be made arbitrarily large over an arbitrarily wide frequency range. A designer is always faced with certain performance trade-offs; these involve command-following and external-force rejection versus stability robustness to high-frequency unmodelled dynamics. The conflict between these two sets of objectives is evident in most positioning systems(2). If one designs a model-based compensator for an unconstrained manipulator, bearing in mind the objectives of disturbance rejection and robustness to model uncertainties, then the closed-loop system will operate according to the specified criteria as long as the manipulator travels inside the unconstrained environment. The system will try to reject all external forces and reach the assigned reference input. However, once the manipulator crosses the boundary of the unconstrained environment (i.e., the manipulator interacts with the environment), the dynamics of the system will change and stability will no longer be guaranteed with the same controller. In fact, the system is now likely to become unstable. Even if stability is preserved, large contact forces may result. Once the manipulator is constrained, the compensator treats the interaction forces as disturbances and tries to **reject** them, thus causing more interaction forces and torques. Saturation, instability, and physical failure are the consequences of this type of interaction. But, in many applications such external forces should be <u>accommodated</u> rather than resisted.

<u>An alternative to external-force rejection arises if it is possible to specify the interaction forces generated in response to imposed position.</u> The design objective is to provide a stabilizing dynamic compensator for the system such that the ratio of the position of the closed-loop system to an interaction force is constant within a given operating frequency range. The above statement can be mathematically expressed by equation 1.

$$\delta D(j\omega) = K\,\delta Y(j\omega) \qquad \text{for all } 0<\omega<\omega_0 \qquad (1)$$

where:

$\delta D(j\omega)$ = n×1 vector of deviation of the interaction forces and torques from equilibrium value in the global cartesian coordinate frame.

$\delta Y(j\omega)$ = n×1 vector of deviation of the interaction -port position and orientation from an equilibrium point

* In this paper force implies force and torque and position implies position and orientation.

in the global cartesian coordinate frame.

K = n×n real-valued non-singular stiffness matrix with constant members.

ω_0 = bandwidth (frequency range of operation)

j = complex number notation, $\sqrt{-1}$

The stiffness matrix (19) is the designer's choice that, depending on the application, contains different values for each direction. By specifying K, the designer governs the behavior of the system in constrained maneuvers. Large members of the K-matrix imply large interaction forces and torques. Small members of the K-matrix allow for a considerable amount of motion in the system in response to interaction forces and torques. Stiffness values, in one sense, represent the type of behavior a designer may wish a stable positioning system to exhibit. For example, if the system is expected to encounter some physical constraint in a particular direction, a stiffness value may be selected such that the desired contact force is ensured in that direction; in directions in which the system is not likely to meet any physical constraints, a stiffness value with a proper position set-point must be selected such that the system follows the desired reference input. Therefore, a K-matrix can be formed to contain stiffness values appropriate for different directions. Even though a diagonal stiffness matrix is appealing for the purpose of static uncoupling, the K-matrix is not restricted to any structure at this stage. Selection of the K-matrix is considered as the first item of the set of performance specifications.

The system must also reject the disturbances (if there are any). If disturbances (e.g., force measurement noise) and interaction forces both contain the same frequency range (or even if the frequency spectra of both signals overlap), then the system in general cannot differentiate between disturbances and the interaction forces. Here we assume that all undesirable disturbances and force measurement noise act on the system at a frequency greater than ω_0 (see the grinding example). An analogy can be observed in tracking systems; if measurement noise and reference input share some frequency spectrum, the system will follow the noise as well as the reference input. The reference input must contain components with frequency spectra much smaller than the spectrum of the measurement noise.

Mechanical systems are not generally responsive to external forces at high frequencies; as the frequency increases, the effect of the feedback disappears gradually, depending on the type of controller used, until the inertia of the system dominates its overall motion Therefore, depending on the dynamics of the system, equation 1 may not hold for a wide frequency range. It is necessary to consider the specification of ω_0 as the second item of the set of performance specifications. In other words, two independent issues are addressed by equation 1: first, a simple relationship between $\delta D(j\omega)$ and $\delta Y(j\omega)$; second, the frequency range of operation, ω_0, such that equation 1 holds true. Besides choosing an appropriate stiffness matrix, K, and a viable ω_0, a designer must also guarantee the stability of the closed-loop system. Therefore, stability is considered to be the third item of the performance specifications.

The stiffness matrix, K, the frequency range of operation, ω_0, and the stability of the closed-loop system, form the set of performance specifications. Note that this set of performance specifications (stiffness, frequency range of operation, and stability) is just a contemporary and practical way of formulating the properties that will enable the closed-loop system to handle constrained maneuvers. The achievement of the set of performance specifications is not trivial; the stiffness of the system cannot be shaped arbitrarily over an arbitrary frequency range. A designer must accept a certain trade-off between performance specifications and stability robustness to model uncertainties. The conflict between the performance specifications and stability robustness specifications is evident in most closed-loop control systems. The set of performance specifications and stability robustness specifications taken together establish a complete set of controller design specifications. Figure 3.1 shows how this set is categorized.

Controller Design Specifications
- 1) Performance specifications
  - 1) Stiffness matrix, K
  - 2) Bandwidth, ω_0
  - 3) Closed-Loop stability
- 2) Stability Robustness Specifications

Figure 3-1: Controller Design Specifications

Establishing the set of performance specifications (K, ω_0 and stability) gives designers a chance to express (at least to themselves) what they wish to have happen during a constrained manipulation via a manipulator. Note that the set of performance specifications does not imply any choice of control techniques. We have not even said how one might achieve the set of performance specifications. Such a set only allows designers to translate their objectives (after understanding the mechanics of the problem) into a form that is meaningful from the standpoint of control theory.

3.1 Performance Specifications

We are looking for a mathematical model that will enable us to parameterize the three items of the set of performance specifications (K, ω_0 and stability). The parameterization must allow the designer to specify the stiffness matrix, K, and the frequency range of operation, ω_0, independently, while guaranteeing stability. All such performance specifications can be mathematically expressed by equation 2.

$$\delta D(s) = (K + Cs + Js^2)\ \delta Y(s),\ s = j\omega \ \text{ for all } 0 < \omega < \omega_0 \qquad (2)$$

$(K + Cs + Js^2)$ = impedance

K, C and J are n x n real-valued non-singular matrices. We use the Laplace operators in equation 2, to emphasize that the entire set of performance specifications can be shown by a linear dynamic equation in the time domain (see section 6). Proper selection of the K-matrix allows the designer to express the desired stiffness, while judicious choice of the inertia matrix, J, and the damping matrix, C, assures the achievement of ω_o and the stability of the system. To clarify the contributions of J, C and K, consider figure 3-2, the plot of $\delta Y(j\omega)/\delta D(j\omega)$ from equation 2 when n = 1 and the system is slightly underdamped. $\delta Y(j\omega)/\delta D(j\omega)$ remains very close to 1/K for some bounded frequency range. In other words, the plot of $\delta Y(j\omega)/\delta D(j\omega)$ approximately exhibits the relationship in equation 1 for some bounded frequency range.

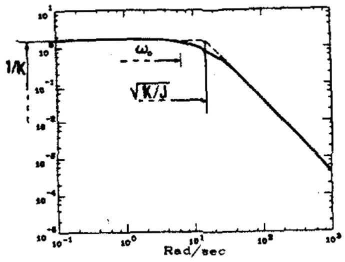

Figure 3-2: Plot of $\delta Y(j\omega)/\delta D(j\omega)$ when n=1

Therefore, K in equation 2 parameterizes the first item of the set of performance specifications. Let the frequency range for which inequality 3 is true be ω_0.

$$|Js^2 + Cs| < \beta |K| \qquad s = j\omega, \quad n=1 \tag{3}$$

where β is a positive number less than one which measures how close the proposed impedance is to K. Note that our only purpose in introducting β is to say that for the bounded frequency range $(0,\omega_0)$, the impedance in equation 2 behaves approximately like the K-matrix. β represents this approximation and is not a design parameter. If K is given, then ω_0 and the stability of the system (the second and third items of the set of performance specifications, as given by equation 2), depend on J and C. In other words, the designer can change either J or C to affect ω_0 and the stability of the system. For example, for a given K and C, decreasing J causes the corner frequency, $\sqrt{K/J}$, and consequently ω_o, to increase. Changing J also moves the eigenvalues of the system. For a given positive set of K and C, a negative J locates one eigenvalue in the right-half complex plane, while a positive J guarantees that both eigenvalues stay in the left-half complex plane. The dependence of ω_0 and the stability of the system on C can be investigated in a similar way. Because of the dependence of ω_0 and the stability of equation 2 on J and C, it can be shown that for a given K, there exist many J and C such that two eigenvalues of the system are always in the left-half complex plane and $\delta Y(j\omega)/\delta D(j\omega)$ remains arbitrarily close to 1/K for all $0<\omega<\omega_0$. We consider J and C as two factors that parameterize the second and third items of the performance specifications. If we consider C *as a parameter that only guarantees a stable and slightly* over-damped (or slightly under-damped) system, then we can claim that J is the only effective parameter in increasing or decreasing the frequency range of operation, ω_0, for a given K. Since a heavy system is always slower than a light system, a large target inertia, J, implies a slow system (narrow ω_o), while a small target inertia implies a fast system (wide ω_0).

The parameterization of the set of performance specifications in the case of more than one dimension is similar to the case of when n = 1. Matrix K in equation 2 models the first item of the set of performnce specifications because the behavior of $(K+Cs+Js^2)$ approximates that of K for some bounded frequency range. It can be shown that for a given matrix K there exist many J and C matrices such that equation 2 offers a stable impedance and $(K+Cs+Js^2)$ is close to K for all $0<\omega<\omega_0$. For example, if J and C are selected to be $\gamma_1 K$ and $\gamma_2 K$ (where γ_1 and γ_2 are scalars), then the characteristic equation of equation 2 yields n uncoupled second-order equation for the eigenvalues of the system. γ_1 and γ_2 can be selected such that all eigenvalues are in the left-half complex plane. The smaller γ_1 is selected to be, the wider ω_0 will be. Of course, this may not be the best way of choosing J and C, but it does show that there exist many J and C matrices such that with a proper K, equation 2 models all three items of the set of performance specifications. Again, if we consider matrix C as a parameter that only guarantees a stable and slightly over-damped (or slightly under-damped) system, then we can claim that matrix J is the only effective parameter in *increasing or decreasing the frequency range of operation*, ω_0, for a given K-matrix. The following is a summary of the parameterization of the set of performance specifications:

stiffness matrix ....> K;
ω_o ....> J;
stability ....> C.

At this stage, we do not restrain matrices J, C and K to any structure. The only restriction is that J, C and K be non-singular matrices.

Equation 2 is not the only possible parameterization of the performance specifications. Similarity of the natural behavior of manipulators to the form introduced by equation 2 is one reason for the choice of the second-order impedance. Within some bounded frequency range, manipulator dynamics are governed by Newton's equations, which are of second order for each degree of freedom. Practitioners often observe an attenuation in frequency response tests on manipulators for some bounded frequency range which can be approximated 40db per decade. At high frequencies, other dynamics contribute to the dynamic behavior of manipulators. We chose a second-order impedance because of this dynamic similarity.

Section 5 and 6 explain some properties of the second-order impedances. Throughout this paper, equation 2 is referred to as the target dynamics. Other forms of this equation are presented in Section 6.

Here, we use an example to illustrate a potential difficulty with matrix J. Consider a diagonal K-matrix. If K is chosen so that $K=diag(k_1,k_2,...,k_n)$, then $J=diag(j_1,j_2,...,j_n)$ and $C=diag(c_1,c_2,...,c_n)$ can be selected to guarantee that each channel has the desired frequency range of operation. Even though K is selected to be a diagonal matrix to ensure uncoupling, there exist an infinite number of J-matrices (not necessarily diagonal) that can guarantee this uncoupling for the desired frequency range of operation. This is true because Js^2 is effective only at high frequencies ($\omega > \omega_0$); for all $0<\omega<\omega_0$, K plays the most important role in determining the response of the system. The size of J is important, not its structure. Of course, the diagonal structure for J makes its selection much easier. As stated earlier, $[K+Cs+Js^2]$ remains very close to K for some bounded frequency range, $0<\omega<\omega_0$. For all $0<\omega<\omega_0$, $(K+Cs+Js^2)$ behaves approximately like K, and the contact forces that are generated in response to those components of the imposed position $\delta Y(j\omega)$ that live in the operating region $0<\omega<\omega_0$ are approximately equal to $K\delta Y(j\omega)$, which is nearly independent of J. (Of course, the response of the system outside the frequency range of operation $(\omega_0<\omega<\infty)$ depends on J.) On the other hand, ω_0 establishes the frequency range in which the size of K is much larger than Js^2. Dependence of ω_0 on the size of J and the independence of the system's response from J, show that the size of J is important and not its structure. (One can consider the size of the J-matrix in terms of its singular values.) A diagonal or a non-diagonal J is equally suitable for an impedance as long as the size of the matrix guarantees that $\delta D(j\omega) \approx K\delta Y(j\omega)$ for all $0<\omega<\omega_0$. In Part 2 of this paper (Reference 24) we will arrive at a non-diagonal J, which can guarantee an uncoupled stiffness for $0<\omega<\omega_0$ without any force measurements. See Part 2 (reference 24) of this paper for a discussion of the selection of the J-matrix.

Some Comments: By specifying the matrices J, C and K, a designer can modulate the impedance of the system. Our primary goal is to achieve a certain set of ω_0 and K in a stable sense. Equation 2 happened to be a suitable target dynamics that can model our superior performance specifications (K, ω_0 and stability). We do not consider J, C and K as our primary performance specifications. J, C and K have $3n^2$ members and arriving at some values for all members of matrices J, C and K independently, without paying attention to their affects on ω_0 and K is an unnecessary overspecification of a peformance criteria.

If a manipulator is in contact with its environment and a new reference point is commanded (e.g., by a supervisory program), then, since the parameters of the impedance in Equation 2 are under control, the resulting interaction force on the system will also be under control. This means that the controlled manipulator will behave like a system that accepts a set of position and orientation commands and reflects a set of forces and torques as output. This is the fundamental characteristic of our method. In other words, this method always allows for closed-loop positioning capabilities. (See Figure 5-1). Stiffness control (19) also offers this characteristic. By assigning different position and orientation commands and by maintaining complete control in equation 2, a designer can achieve the desired contact forces and torques. Note that we still have a positioning system for the manipulator with the ability to modulate the impedance of the system. The terminology "Impedance Control" (10) which is used in literature is incorrect and misleading from the standpoint of control theory (even though it has been widely used). We do not control the impedance, we still control the position and orientation of the manipulator; we only modulate (or adjust) the impedance.

Equation 2 in the time-domain can be described by equation 4.

$$J\ \delta\ddot{Y}(t) + C\ \delta\dot{Y}(t) + K\ \delta Y(t) = \delta D(t) \qquad (4)$$

J, K and C are non-singular matrices. $\delta Y(t)$ and $\delta D(t)$ are $n\times1$ vectors. Even though we use the time-domain representation of the target dynamics in our design method (part 2 of this paper ,(reference24)), we plan to guarantee the achievement of the target dynamics in the frequency domain. We also select the parameters of equation 2 to guarantee the design specifications in the frequency domain. Selection of J, C and K to represent a frequency-domain design specification implies shaping the steady-state behavior of the system in response to all frequency components of the imposed motion command. An alternative approach is to specify J, C and K to represent some design specifications in the time-domain. The time-domain representation of the target dynamics without specifying the time-domain representation of the input does not express an adequate target dynamic model for the manipulators. One must also specify the time-domain representation of the input to the system along the time-domain representation of the target dynamics. For example, it is clear that a manipulator will not behave according to equation 4 in time domain if a 100 hertz periodic force input is imposed on a manipulator. This is true because all of the high frequency structural dynamics of the system will contribute to the motion of the system, and equation 4 will no longer be an adequate representation of the target dynamics of the system. In many manipulators the dynamics of the system cannot be shaped in the time domain as equation 4 even for a step input force function. Note that the condition, $0<\omega<\omega_0$, in equation 2 assigns a restriction on the components of the inputs to the system. In other words, we do specify the type of input for the proposed impedance in equation 2.

3.2 Stability Robustness Specifications

The stability robustness specifications arise from the existence of model uncertainties (16, 12). The model uncertainties fall into two classes. Lack of exact knowledge about the parameters of the modelled dynamics (e.g., the inertia matrix) constitute the first class of model uncertainties. High frequency unmodelled dynamics (such as bending or torsion dynamics of the members) form the second class of unmodelled dynamics. Note that the model uncertainties of the second class generally give rise to modelling errors only at high frequencies, while the model uncertainties of the first class can contribute to modelling errors at all frequencies. If the compensated system does not satisfy the stability robustness specifications, the system may not become unstable. This is true because our robustness test is a sufficient condition for stability. Satisfaction of the robustness test guarantees stability, while the failure of the robustness test does not necessarily imply instability. If one cannot meet the stability robustness specifications at high frequencies, it is necessary to consider the higher-order dynamics (if at all possible) when modelling the system. Adding the higher-order dynamics to the system allows for weaker stability robustness specifications at high frequencies. If higher-order dynamics cannot be determined, it is **necessary to** compromise on the set of performance specifications. A small ω_0 will allow designers to meet strong sets of stability robustness specifications at high frequencies. On the other hand, with a very small ω_0, stability robustness to parameter uncertainties may not be satisfied. This is true because stability robustness to parameter uncertainties assigns a lower bound on ω_0. To achieve a wide ω_0, a designer should have a good model of the manipulator at high frequencies (and consequently, a weak set of stability robustness specifications at high frequencies). Because of the conflict between desired ω_0 and stability robustness to high-frequency dynamics, it is a struggle to meet both sets of specifications for a given model uncertainty. The frequency range of operation, ω_0, cannot be selected to be arbitrarily wide if a good model of the manipulator does not exist at high frequencies, while a good model of the manipulator at high frequencies makes it possible to retain the target dynamics for a wide ω_0. The relationship between ω_0 and stability robustness will be presented in Part 2 (Reference 24) of this paper. Even though ω_0 is the major candidate that can be used to compromise against stability robustness specifications there are other freedoms in design technique that sometimes can be used for the same purpose. This will be clarified in Part 2 of this paper.

4. Examples of Applications of Compliant Motion

The following examples illustrate some applications of impedance control. For the purpose of understanding the application of this theory, the problems in these examples are simplified.

Grinding. Consider the grinding of a surface by a manipulator; the objective is to use the manipulator to smooth the surface down to the commanded trajectory represented by the dashed line (12) in Figure 4-1. Here we give an approach in which this task is performed by a manipulator. It is intuitive to design a closed-loop positioning system for the manipulator with a large stiffness value in the R-direction and a low stiffness value in the T-direction.

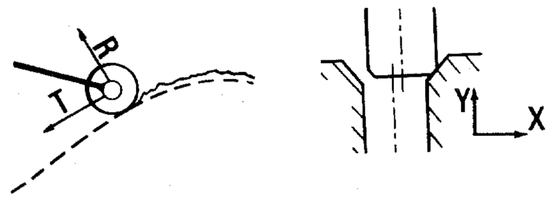

Figure 4-1 a: Grinding a Surface via a Manipulator, b: Peg in Hole

In many tasks, it is beneficial to produce the compliant motion in an active end-effector with a few degrees of freedom instead of producing the compliant motion for the entire arm. A large stiffness value in the R-direction causes the end-point of the manipulator to reject the external forces and stay very close to the commanded trajectory (dashed line). The larger the stiffness of the manipulator in the R-direction, the smoother the surface will be. Given the volume of metal to be removed, the desired tolerance in the R-direction prescribes an approximate value for stiffness in the R-direction. The force necessary to cut in the T-direction at a constant traverse speed is approximately proportional to the volume of metal to be removed (1). Therefore, the larger the "bumps" on the surface, the slower the manipulator's end-point must move in the T-direction. This is necessary because the slower speed of the end-point along the surface implies less volume of metal to be removed per unit of time, and consequently, less force in the T-direction. To remove the metal from the surface, the manipulator should slow down in response to external forces resulting from large "bumps." The above explanation means that it is necessary for the manipulator to accommodate the interaction forces along the T-direction, which directly implies a small stiffness value in the T-direction. If a designer does not accommodate the interaction forces by specifying a small stiffness value in the T-direction, then the large "bumps" on the surface will produce large contact forces in the T-direction. Two problems are associated with large contact forces in the T-direction: the cutting tool may stall (if it does not break); a slight motion may develop in the manipulator's end-point motion along the R-direction, which might exceed the desired tolerance. A small value for stiffness in the T-direction (relative to the stiffness in the R-direction) guarantees the desired contact forces in the T-direction. The rougher the surface is, the smaller K must be in the T-direction. The frequency spectrum of the roughness of the surface and the desired translational speed of the manipulator end-point along the surface determine the frequency range of operation, ω_0. Given the stiffness in both directions, a designer can arrive at proper values for J and C to guarantee ω_0 and stability. At each point on the trajectory, a controller must be modified in the joint-angle coordinate frame such that the desired target impedance of the form in Equation 2 is achieved in the global coordinate frame. The rotation of the cutter causes some high-frequency disturbances in the manipulator. The contact force measurement is also noisy. ω_0 must be selected to be lower than the frequency range of the cutter disturbances and the force measurement noise. The satisfaction of equation 2 prevents the system from responding to these disturbances and noises.

Peg-in-Hole. The peg-in-hole task is generic to many assembly operations such as inserting a rod into a hole or a computer board into a slot. There are many strategies for this task (for an example see reference [22]); most assume that the manipulators are capable of producing compliant motion. We are not giving a complete solution to the peg-in-hole problem, but just a simplified example to illustrate the use of this method in such maneuvers. Once the peg is located at the position shown in Figure 4-1-b, a small stiffness in the X-direction must be selected. If there is any misalignment between the peg axis and the hole axis, a small stiffness in the X-direction causes the manipulator end-point to align itself with the axis of the hole. If the stiffness in the X-direction is large, the manipulator end-point will not move in the X-direction, and large contact forces will result. A large stiffness must be selected for the Y-direction to guarantee a positioning system that will reject the friction forces in the Y-direction and insert the peg into the hole

5. Global Stability of the Target Dynamics

We consider two issues of importance in analyzing the stability of the manipulator that interacts with the environment. The first issue concerns the condition under which equation 2 offers a stable target dynamics. The stability of the target dynamics is not enough to assure the stability of the manipulator and its environment taken as a whole. This brings up the second issue: the global stability of the manipulator and its environment.

The target dynamics of a manipulator must be stable. Note that stability is not a condition for achievability. (See Part 2 of this paper (Reference 24).) Stability of the target dynamics depends on the values of J, C and K. One sufficient condition for the stability of the target dynamics is given by Theorem 1 (3, 4).

Theorem 1. If J, C and K are real and symmetric, positive definite matrices, then the system in equation 2 is stable, and if C and K are symmetric, non-negative definite matrices, then the system in equation 2 will be marginally stable. If K and/or C are symmetric, positive, semi-definite matrices, then some or all eigenvalues will be on the imaginary axis. (These cases are considered unstable.) Note that the conditions on J, C and K are sufficient for stability, but not necessary. We might arrive at a set of J, C and K that assures stability without satisfying the theorem condition. The stability of the target dynamics is not enough to assure stability of the overall system of the manipulator and its environment. In other words, the following question cannot be answered by this theorem: If a manipulator with a stable impedance as expressed by equation 2 is in contact with a stable environment, does the system of the manipulator and its environment remain stable? This is not clear; two stable systems interacting with each other may result in an unstable system. The following theorem is needed for the rigorous assurance of the overall stability of the manipulator and its environment:

Theorem 2. If the closed-loop dynamic behavior of the manipulator is given by equation 5:

$$J\,\delta\ddot{Y}(t)+C\,\delta\dot{Y}(t)+K\delta Y(t)=\delta D(t),\quad \delta D(t) \text{ and } \delta Y(t)\in R^n \qquad (5)$$
$$J=J^T>0, \qquad K=K^T>0, \qquad C=C^T>0;$$

and if the environment is a system with the dynamic behavior represented by equation 6:

$$J_e\,\delta\ddot{Y}_e(t)+C_e\,\delta\dot{Y}_e(t)+K_e\delta Y_e(t)=\delta D_e(t)+\delta D_e^o(t) \qquad (6)$$

$\delta D_e(t)$, $\delta D_e^o(t)$ and $\delta Y_e(t)\in R^m$

$J_e=J_e^T>0$, $K_e=K_e^T>0$, $C_e=C_e^T>0$;

where:

$\delta D_e(t)$ = the force that the manipulator exerts on the environment;

$\delta D_e^o(t)$ = all other foces on the environment (uncorrelated with the manipulator states and environment states); and

$\delta D(t)$ = the environmental force on the manipulator,

then the overall system (manipulator and environment) is stable. The Proof is given in Appendix I.

According to this theorem, if J, C and K are selected as symmetric, positive definite matrices, the overall system of the manipulator and its environment taken together will yield eigenvalues in the left complex plane. Note that this theorem guarantees the global stability of the manipulator and the environment taken as a whole, if the manipulator behaves according to equation 4. If the controller does not achieve the target impedance exactly, but results in a controlled

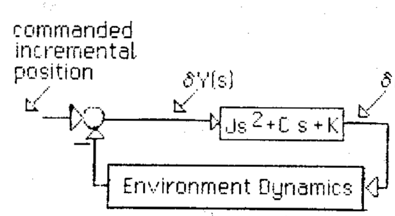

Figure 5-1: The interaction of the manipulator and the environment in the ideal case when the target impedance is achieved for all $0<\omega<\infty$.

behavior "approximately" like the target dynamics for a bounded frequency range, then the above theorem does not guarantee the global stability. The importance of theorem 2 is that it shows that the target impedance has desirable properties. The block diagram in Figure 5-1 shows how the manipulator and the environment interact with each other in an ideal case when the target impedance is achieved for all $0<\omega<\infty$. $\delta Y(s)$ is the imposed position on the manipulator which consists of the algebraic addition of the commanded incremental position and the environmental position.

6. Geometric Properties of the Target Dynamics

Since a geometric approach is being considered for compensator design (Part 2 of this paper (Reference 24)), it is necessary to identify the eigenstructure properties of the target dynamics. The target dynamics that correlate interaction forces with system position are given in state-space form by equations 7 and 8.

$$\begin{pmatrix} \delta\dot{Y}(t) \\ \delta\ddot{Y}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 0_{nn} & I_{nn} \\ -J^{-1}K & -J^{-1}C \end{pmatrix}}_{A_t} \begin{pmatrix} \delta Y(t) \\ \delta\dot{Y}(t) \end{pmatrix} + \underbrace{\begin{pmatrix} 0_{nn} \\ J^{-1} \end{pmatrix}}_{B_t} \delta D(t) \qquad (7)$$

$$\delta Y(t) = \underbrace{\begin{pmatrix} I_{nn} & 0_{nn} \end{pmatrix}}_{C_t} \begin{pmatrix} \delta Y(t) \\ \delta\dot{Y}(t) \end{pmatrix} \qquad (8)$$

$A_t = 2n\times 2n$, $B_t = 2n\times n$, $C_t = n\times 2n$

and also: $G_t(s) = (J s^2 + C s + K)^{-1} = C_t (sI_{2n2n} - A_t)^{-1} B_t$

$G_t(s)$ is the transfer function matrix that maps the interaction force to the end point position. ($G_t(s)$ is the desired target transfer function matrix.) The advantage of this form is that it enables a designer to describe the target dynamics of a system in geometrical terms. A_t contains information concerning the modes (eigenvalues) and the relative distribution of the modes (eigenvectors) among the states. The target dynamics in equation 2 imply a closed-loop behavior for the manipulator in the frequency domain. Our goal is to make the manipulator behave according to equation 2 for all $0<\omega<\omega_0$. Note that in general the closed-loop behavior of a system cannot be shaped arbitrarily over an arbitrary frequency range. The time-domain presentation of the target dynamics in equation 7 and 8 offer a set of eigenvalues and eigenvectors to model the internal dynamic behavior of the target dynamics. Each eigenvalue of the target impedance, λ_i, and its corresponding right eigenvector, z_i, can be computed from equation 9.

$$(\lambda_i I_{2n2n} - A_t) z_i = 0_{2n}, \quad z_i \neq 0_{2n}, \quad i = 1, 2, ..., 2n \qquad (9)$$

Substituting for A_t from equation 7 in equation 9 and solving for z_i results in equation 10

$$z_i = \begin{pmatrix} q_i \\ \lambda_i q_i \end{pmatrix} \qquad (10)$$

$$\text{where: } (J \lambda_i^2 + C\lambda_i + K) q_i = 0_n, \quad q_i \neq 0_n, \quad i = 1, 2, ..., 2n \qquad (11)$$

The impedances that always yield a complete set of eigenvectors are called simple (4, 13,11) *. In other words, simple impedances guarantee a complete set of eigenvectors, despite the multiplicity of their eigenvalues. Equations 7 and 8 represent a state-space relationship between end-point position and interaction force in the global coordinate frame. The transformation of the end-point position from the global coordinate frame to the joint-angle coordinate frame is given in reference (21); it results in the following equation:

$$\delta Y(t) = J_c \, \delta\theta(t) \qquad (12)$$

where J_c is the Jacobian of the matrix that transforms joint-angle coordinates to global coordinates. Equation 2 represents a dynamic behavior in the neighborhood of an equilibrium point; $\delta D(t)$ and $\delta Y(t)$ are small increments away from an equilibrium point (a point with zero speed in space). Knowing this, we can write:

$$\delta\dot{Y}(t) = J_c \, \delta\dot{\theta}(t) \qquad (13)$$

If v_i is the right eigenvector of the target dynamics in the joint-angle coordinate frame, then:

$$v_i = \begin{pmatrix} J_c^{-1} & 0_{nn} \\ 0_{nn} & J_c^{-1} \end{pmatrix} z_i = \begin{pmatrix} J_c^{-1} q_i \\ \lambda_i J_c^{-1} q_i \end{pmatrix} \quad J_o \text{ is non-singular}$$

the 2n eigenvectors of equation 14 form a 2n x 2n matrix V:

$$V = [v_1 \; v_2 \; .....v_{2n}]. \qquad (15)$$

V is a basis for the state-space representation of the target dynamics in the joint-angle coordinate frame. V shows how the desired modes are coupled among the states of the target dynamics. The 2n eigenvalues resulting from equations 9 or 10 are invariant under any linear transformation and form a self-conjugate constant set $\Lambda=\{\lambda_i: i=1,2,..., 2n\}$. Λ and V taken together describe the eigenstructure of the desired impedance in the joint-angle coordinate frame. For simple impedances, V is a full rank matrix.

* If $(J\lambda^2+C\lambda+K)$ has degeneracy equal to the multiplicity of eigenvalue λ_α in equation 11, $(J\lambda^2+C\lambda+K)$ is a simple impedance. If J, C and K are symmetric positive definite matrices, then $(J\lambda^2+C\lambda+K)$ is a simple impedance.(4,13,11)

7. Conclusion

We started with conventional controller design specifications concerning the treatment of the interaction forces and torques when the manipulator is not constrained. Generalizing this treatment to include cases when the manipulator is constrained, we stated a set of controller design specifications to assure compliant motion with stability in the presence of bounded uncertainties (Figure 3-1). One of the most important contributions of this paper is the formulation of the concept of compliant motion in terms of a meaningful set of controller design specifications. This set (shown in Figure 3-1) is a proper definition of the compliant motion. Equation 2 can parameterize our set of performance specifications. The following is a summary of the parameterization of the set of performance specifications:

stiffness matrix ....> K;
ω_0 ....> J;
stability ....> C.

We assume C to be a matrix that always produces a slightly overdamped or underdamped stable system; therefore, for a given K-matrix, the J-matrix is the parameter that affects ω_0 the most, and many J-matrices can parameterize ω_0. In particular, we claim (and we will show in Part 2 (Reference 24)) that a wide ω_0 (or a small J-matrix) may cause instability in the presence of high frequency unmodelled dynamics. The stability of the target impedance and its global stability with the environment result from the appropriate choice of the target dynamics and not the design methodology.

I. Appendix I

Proof: Since the manipulator is in contact with the environment, vectors $\delta Y(t)$ and $\delta Y_e(t)$ might have members in common. Form a p-dimensional vector $\delta W(t)$ such that equation 16 and equation 17 are satisfied $(n + m \geqslant p)$. $\delta W(t)$ is a vector that contains all states of the manipulator and environment. Since the manipulator and environment are in contact with each other, $\delta Y_e t)$ and $\delta Y(t)$ will have some common members. The first (p-n) members of $\delta W(t)$ are those states of the environment that are not states of the manipulator. The last (p-m) members of $\delta W(t)$ are those states of the manipulator that do not represent the environmental dynamics.

$$\delta Y_e(t) = T_e\, \delta W(t) \qquad (16)$$

$$\delta Y(t) = T_y\, \delta W(t) \qquad (17)$$

T_e and T_y are m x p and n x p matrices with 0 and 1 as their members. Substituting for $\delta Y_e(t)$ and $\delta Y(t)$ in equations 5 and 6 results in equations 18 and 19.

$$J\, T_y\, \delta \ddot{W}(t) + C\, T_y\, \delta \dot{W}(t) + K\, T_y\, \delta W(t) = \delta D(t) \qquad (18)$$

$$J_e T_e \delta \ddot{W}(t) + C_e T_e\, \delta \dot{W}(t) + K_e T_e\, \delta W(t) = \delta D_e(t) + \delta D_e^o(t) \qquad (19)$$

Because of the interaction between the manipulator and the environment, equation 20 is also true.

$$T_y^T\, \delta D(t) = -T_e^T\, \delta D_e(t) \qquad (20)$$

Omitting $\delta D(t)$ and $\delta D_e(t)$ from equations 18 and 19 by means of equation 20 results in:

$$(T_y^T J\, T_y + T_e^T J_e\, T_e)\, \delta \ddot{W}(t) + (T_y^T C\, T_y + T_e^T C_e\, T_e) \delta \dot{W}(t) +$$
$$(T_y^T K\, T_y + T_e^T K_e\, T_e)\, \delta W(t) = T_e^T\, \delta D_e^o(t) \qquad (21)$$

It can be verified that:

$(T_y^T J\, T_y + T_e^T J_e\, T_e)$ = a symmetric, positive definite matrix;
$(T_y^T C\, T_y + T_e^T C_e\, T_e)$= a symmetric, positive definite matrix;
$(T_y^T K\, T_y + T_e^T K_e\, T_e)$= a symmetric, positive definite matrix.

According to Theorem 1, equation 21 (which shows the dynamics of the manipulator and the environment) is stable. According to this theorem, if J, C and K are selected as symmetric, positive definite matrices, the overall system of the manipulator and its environment taken together will yield eigenvalues in the left complex plane.

Note that this theorem guarantees the global stability of the manipulator and the environment taken as a whole, if the manipulator behaves according to equation 4. The block diagram in Figure I-1 shows how the manipulator and the environment interact with each other in an ideal case when the target impedance is achieved for all $0<\omega<\infty$. $\delta Y(s)$ is the imposed position on the manipulator which consists of the algebraic addition of the commanded incremental position and the environmental position. If the controller does not achieve the target impedance exactly, but results in a controlled behavior "approximately" like the target dynamics for a bounded frequency range, then the above theorem does not guarantee the global stability. The importance of Theorem 2 is that it shows that the target impedance has desirable properties. The resulting impedance when the target dynamics is achieved only for a bounded frequency range can be shown as:

$$(J\, s^2 + Cs + K\,)(\, I_{nn} + E_t(s)\,) \qquad (22)$$

where $E_t(s)$ shows the difference between the achievable target dynamics and the ideal target dynamics. The closed-loop combination of the manipulator and the environment, considering expression 22, is shown in figure I-2. The global stability of the system in Figure I-2 is no longer guaranteed by this theorem. Using the result in references (14,18), the closed-loop system in Figure I-2 will be stable if the inequality 23 is satisfied for all $0<\omega<\infty$.

$$\sigma_{min}\{\, I_{nn} + (Js^2 + Cs + K)^{-1}\, (T_y\, G_e(s) T_y^T)^{-1}\,\} > \sigma_{max}\,(E_t(s)) \quad (23)$$

where: $G_e(s) = (T_e^T J_e T_e\, s^2 + T_e^T\, C_e\, T_e\, s + T_e^T K_e T_e)^{-1}$

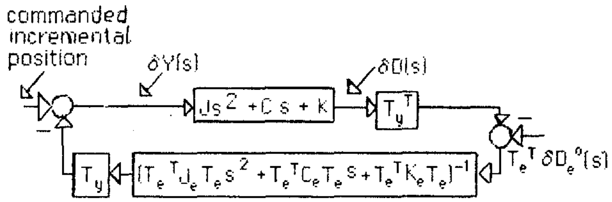

Figure I-1 The interaction of the manipulator and environment in the ideal case when the target impedance is achieved for all $0 < \omega < \infty$.

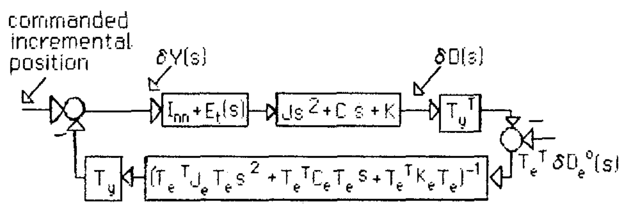

Figure I-2: The interaction of the manipulator and the environment when the target impedance is achieved for some bounded frequency range.

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